Complexity of Target for pushdown RBN

Context

Reconfigurable Broadcast Networks (RBN) are a model for large groups of identical agents communicating via unreliable broadcast. A pushdown RBN (PRBN) is simply one where each agent is modelled by a pushdown transition system.

A PRBN is described by a tuple $(Q, M, \Gamma, \Delta, q_0)$ where Q is a finite set of states, M a finite alphabet of messages, Γ a finite stack alphabet with a special letter $\perp$, q_0 an initial state, and

$$\Delta \subseteq Q \times \{\mathbf{br}(m), \mathbf{rec}(m) \mid m \in M\} \times \{push(\gamma), pop(\gamma), nop \mid \gamma \in \Gamma\} \times Q$$

a transition function.

A run starts with an arbitrarily large set of agents, all in state q_0. A step consists of one agent executing a transition with a broadcast $\mathbf{br}(m)$, and an arbitrary subset of other agents executing a transition with a matching reception $\mathbf{rec}(m)$.

Formally, a local configuration is a pair $(q, \sigma) \in Q \times \Gamma^*$. A local step $(q, \sigma) \xrightarrow{op_1} (q', \sigma')$ is defined when there exists a transition (q, op_1, op_2, q') such that either:

- $op_2 = push(\gamma)$ and $\sigma' = \gamma\sigma$
- $op_2 = pop(\gamma)$ and $\sigma = \gamma\sigma'$
- $op_2 = nop$ and $\sigma' = \sigma$

A configuration is a function from a finite non-empty set of agents $\mathbb{A}$ to the set of local configurations $C : \mathbb{A} \to Q \times \Gamma^*$. An initial configuration is such that $C(a) = (q_0, \perp)$ for all $a \in \mathbb{A}$. A step $C \to C'$ is defined when there exists an agent $a_{\mathbf{br}}$ such that there is a local step $C(a_{\mathbf{br}}) \xrightarrow{\mathbf{br}(m)} C'(a_{\mathbf{br}})$ and for all $a \neq a_{\mathbf{br}}$, either

- there is a local step $C(a) \xrightarrow{\mathbf{rec}(m)} C'(a)$, or
- $C(a) = C'(a)$

A run is a sequence of consecutive steps, it is initial if it starts in an initial configuration.

Classical problems on such models ask whether a given set of configurations is reachable. A particular case of interest is the reachability of a configuration where all agents are in a given subset of states $S \subseteq Q$. This problem is called Target.

The open problem is to find tight complexity bounds on the Target problem for PRBN. The problem on fnite-state RBN is known to be solvable in polynomial time. An easy reduction from Horn satisfiability shows PTIME-completeness.

On the other hand, one can show that the problem for PRBN is in NP: to witness Target, one can guess a set of messages $M' \subseteq M$ of messages used, and two orders on this set, an order of appearance $\leq_a$ and one of disappearance $\leq_d$.

It suffices to check that for all $m \in M'$ there exists a run $(q_0, \bot) \xrightarrow{*} (q, \sigma) \xrightarrow{*} (q_f, \sigma_f)$ only receiving messages smaller than m for $\leq_a$ on the first part and messages from M' on the second part, as well as a run $(q_0, \bot) \xrightarrow{*} (q', \sigma') \xrightarrow{*} (q_f, \sigma_f)$ only receiving messages from M' on the first part and from $\{m' \in M' \mid m' <_d m\}$ on the second one.

This can be checked in polynomial time.

Credits and related work

RBN were introduced (with finite-state systems) in [2], and the precise complexity of the main problems was analysed more finely in [3]. Pushdown RBN were introduced in [1], with in particular a proof that the coverability problem (whether there is a reachable configuration with at least one agent in a given state q_f) is decidable in PTIME for this model.

References

[1] A. R. Balasubramanian. "Parameterized Verification of Coverability in Well-Structured Broadcast Networks". In: *Proceedings Ninth International Symposium on Games, Automata, Logics, and Formal Verification, GandALF 2018, Saarbrücken, Germany, 26-28th September 2018*. Ed. by Andrea Orlandini and Martin Zimmermann. Vol. 277. EPTCS. 2018, pp. 133–146. DOI: `10.4204/EPTCS.277.10`. URL: `https://doi.org/10.4204/EPTCS.277.10`.

[2] Giorgio Delzanno, Arnaud Sangnier, and Gianluigi Zavattaro. "Parameterized Verification of Ad Hoc Networks". In: *CONCUR 2010 - Concurrency Theory, 21th International Conference, CONCUR 2010, Paris, France, August 31-September 3, 2010. Proceedings*. Ed. by Paul Gastin and François Laroussinie. Vol. 6269. Lecture Notes in Computer Science. Springer, 2010, pp. 313–327. DOI: `10.1007/978-3-642-15375-4\_22`. URL: `https://doi.org/10.1007/978-3-642-15375-4%5C_22`.

[3] Giorgio Delzanno et al. "On the Complexity of Parameterized Reachability in Reconfigurable Broadcast Networks". In: *IARCS Annual Conference on Foundations of Software Technology and Theoretical Computer Science, FSTTCS 2012, December 15-17, 2012, Hyderabad, India*. Ed. by Deepak D'Souza, Telikepalli Kavitha, and Jaikumar Radhakrishnan. Vol. 18. LIPIcs. Schloss Dagstuhl - Leibniz-Zentrum für Informatik, 2012, pp. 289–300. DOI: `10.4230/LIPIcs.FSTTCS.2012.289`. URL: `https://doi.org/10.4230/LIPIcs.FSTTCS.2012.289`.